The Mathematics of Infinity, Discover Hopkins 2014

Basic Course Information:
Course number: AS110.160
Instructor: Vitaly Lorman
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Text: *Everything and More: A Compact History of Infinity*, David Foster Wallace
ISBN-13: 978-0393339284
Available from the university bookstore

Assignments:

The majority of the work for this class will consist of reading, primarily from *A Compact History of Infinity*, but from other sources as well. There will be two problem sets–one on mathematical logic and proof and one on the epsilon-delta definition of limits. Finally, in the last week you will pick a topic related to one of the ideas we've covered in the course and write a short paper on it. I'll say more about this in class.

Course description, topics and themes:

We will begin at the birth of pure mathematics in Ancient Greece. This will immediately lead us into one of the fundamental themes of the course: *the nature of abstraction and its relation to both mathematics and the world*. On the mathematical side, we will complement this discussion with a crash course on logic and what it means to write a mathematical proof.

We will then read about Zeno's paradoxes, through which infinity first enters the story for us. As amateur philosophers, we will attempt to clearly frame Zeno's arguments and discuss the strength of his premises. The problem of providing a rigorous, mathematically and philosophically satisfying, solution to Zeno's paradoxes will be a major theme of the first half of the course. We will develop the necessary mathematics by using our newfound understanding of logic to construct intuitively appealing and rigorous definitions of limits in their many forms. Along the way, we will discuss the difference between actually resolving a paradox and merely defining it away. This relates intimately to what constitutes *good* mathematics: we will attempt to construct a working definition, discussing the notions of usefulness, beauty, simplicity, and coherence.

The above discussion will be framed in a historical context–we will read about how mathematicians and philosophers, from Ancient Greece through the 20th century attempted to make sense of infinity. We will address the question of why it took so long for calculus to be invented and examine early attempts at its development.

Developing a rigorous notion of limits will lead us straight into the heart of the foundations of mathematics via a close examination of the Real Line. Here we will learn about the basic notions of set theory and discuss what is meant by a foundation for mathematics. We will talk about early attempts at foundations and the paradoxes that plague them (specifically, Russell's paradox). We will examine the problems that self-reference and vicious circles cause. Time permitting, we will investigate Godel's theorems, which once and for all put to rest the hope for a groundwork for mathematics which is both consistent (contains no contradictions) and complete (every true statement can be proved).

We will also read about the philosophical side of the above story and discuss questions such as "in what sense do mathematical objects exist?" and "what is mathematical truth?". We will learn about Platonism, Formalism, and Intuitionism, the three major schools of thought on philosophy of mathematics in the early 20th century. A major emphasis will be on the fact that what philosophy of mathematics one believes determines what sort of mathematics one is allowed to do–to some extent, math is not independent from philosophy.